# Optimal Design of DFT Modulated Filter Bank

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**Abstract-** An optimum DFT-modulated filter bank with sharp transition band as a non-linear optimization has been formulated .The design of this filter is very expensive by the standard method ,and hence the frequency response masking(FRM) technique is developed so as to reduce the complexity in the design process. An effective approach is proposed to solve this optimization problem. To illustrate the effectiveness of the proposed method, simulation examples are presented

**Index Terms-** FIR filters ,FRM technique.

## 1. INTRODUCTION

Today's world thrives on information exchange. Hence the need of the day is that the information be protected well enough to be transmitted over a noisy environment multi rate filter banks are driven by emerging new applications.

In multi rate filter banks, the input signals are divided into multiple sub band signals and are processed by the analysis filters. Then, these signals are down sampled by a decimator. The desired signals are then recovered from the processed Sub band signals by the synthesis filters. If the recovered signals are equal to the input signals, except by a scale factor and a delay,then this filter bank is referred to as having the perfect reconstruction(PR) property. The uniformly decimated filter banks with PR property are of great interest in sub band coding[1-4]. Designs based on the PR reconstruction condition typically allow considerable aliasing in each of the sub band signals. However, the sub band processing block may be sensitive to aliasing in the sub band signals. Also, the sub band processing block may distort the aliasing components in the sub band signals in such a way that the effectiveness of alias cancellation is reduced.

In, the design of a filter bank is formulated as a convex programming that is then solved by the semi-definite programming approach. The filter bank is required to have a good selectivity property. Thus, the transition band of the prototype filter is required to be sharp. However, it is well known[16] that the complexity of an FIR filter is inversely proportional to its transition width. Thus, the design of a filter bank with sharp transition band may require the length of the prototype filter to be substantially longer. As a consequence, the corresponding optimization problem may become too large to be solved successfully by existing optimization methods. To overcome this difficulty, the frequency response masking (FRM) technique is introduced for the design.[14,15,17-19]

The FRM technique to the design of oversampled DFT modulated filter banks with sharp transition bands. Many algorithms are available to solve this class of problems. The most common one is the discretization method.[21] In this paper we simulated a DFT filter bank using MATLAB®.

## 2. CONFIGURATION OF FIR FILTER BANK

The configuration of an oversampled NPR DFT-modulated FIR filter bank (i.e. D < M) is depicted in Fig. 1. The input signal X(z) is divided into M channels. In each channel, the signal is filtered by an analysis filter and then decimated by a factor D. After the decimation, the signals $X_m(z)$, m = 0, 1, . . . , M – 1, are interpolated by the same factor D and then filtered by the synthesis filter bank. We can express this process by the following equations

$$X_m(z) = \frac{1}{D} \sum_{d=0}^{D-1} H_m(z^{1/D} W_D^d) X(z^{1/2} W_D^d) \qquad (1)$$

and

$$Y(z) = \sum_{m=0}^{M-1} Y_m(z) G_m(z)$$

$$= \frac{1}{D} \sum_{d=0}^{D-1} X(z W_D^d) \sum_{m=0}^{M-1} H_m(z W_D^d) G_m(z) \qquad (2)$$
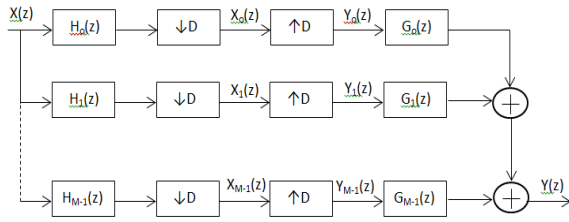
Where $W_D = e^{-j2\pi/D}$ .

Fig.1 M channel D decimated filter bank.

Let the impulse responses of $H_m(z)$ and $G_m(z)$ be $h_m(n)$ and $g_m(n)$, m = 0, 1, . . . , M − 1, n = 0, 1, . . . , $L_p - 1$, respectively. We consider the analysis and synthesis filters that are exponentially modulated by a single real-valued FIR prototype filter p [n] given below

$$h_m[n] = p[n]e^{j(2\pi mn/M)} \tag{3}$$

$$g_m[n] = p[L_p - 1 - n]e^{j(2\pi mn/M)} \tag{4}$$

Let the transfer function of p[n] be P(z). Then, the transfer function of (3) and (4) can be rewritten as

$$H_m(z) = P(zW_M^{-m}) \tag{5}$$

$$G_m(z) = z^{-(L_p-1)}W_M^{-m(L_p-1)}P(z^{-1}W_M^m) \tag{6}$$

The ideal low-pass prototype filter P(z) is given by

$$|P_{ideal}(e^{jw})| = \begin{cases} 1 & if\ 0 \le w \le \pi/M \\ 0 & if\ \pi/M < w \le \pi \end{cases} \tag{7}$$

However, this ideal filter cannot be realised. Thus, we relax the magnitude response of P(z) as follows

$$|P(e^{jw})| = \begin{cases} 1 & if\ 0 \le w \le \dfrac{(1-\rho)\pi}{M} \\ 0 & if\ \dfrac{(1+\rho)\pi}{M} < w \le \pi \end{cases} \tag{8}$$

where $\rho$ is the so-called roll-off factor. Here, we suppose that $((1+\rho)\pi/M) \le (\pi/D)$. Substituting (5) and (6) into (2), we obtain

$$Y(z) = \frac{1}{D}z^{-(L_p-1)}\sum_{d=0}^{D-1}X(zW_D^d)\sum_{m=0}^{M-1}W_M^{-m(L_p-1)}$$

$$\times P(zW_M^{-m}W_D^d)P(z^{-1}W_M^{-m}) \tag{9}$$

$$= \frac{1}{D}z^{-(L_p-1)}\sum_{d=0}^{D-1}\tau_d(z)X(zW_D^d)$$

Where

$$T_d(z) = \sum_{m=0}^{M-1}W_M^{-m(L_p-1)}P(zW_M^{-m}W_D^d)P(z^{-1}W_M^m),$$

$$d = 1.....D-1 \qquad\qquad 10$$

From (8) and D< M, we know that the passband of $P(z^{-1}W_M^m)$ will fall into the stopband of $P(zW_M^{-m}W_D^d)$, d = 1, . . . , D − 1, and vice versa. Thus, the term $P(zW_M^{-m}W_D^d)P(z^{-1}W_M^m)$ be neglected when the stopband ripple of P(z) is very small and hence

$$T_d(z) \quad 0, d = 1.....D-1 \tag{11}$$

Substituting $T_d(z) \quad 0$ into (8), we obtain

$$Y(z) \quad \frac{1}{D}z^{-(L_p-1)}\sum_{m=0}^{M-1}W_M^{-m(L_p-1)}P(z)P(z^{-1})X(z)$$

$$= T_o(z)X(z) \tag{12}$$

Where

$$T_d(z) = \frac{1}{D}z^{-(L_p-1)}\sum_{m=0}^{M-1}W_M^{-m(L_p-1)}P(zW_M^{-m})P(z^{-1}W_M^m) \tag{13}$$

In some applications, (see, for example, [15]), the analysis filter bank and the synthesis filter bank are required to possess a very good frequency selectivity, and hence a sharp transition band of the prototype filter, that is, a small $\rho$. From [16], we see that the length of the optimal prototype filter is inversely proportional to its transition width. In other words, to achieve a sharper transition band, a longer length of the filter is needed. Thus, the design of a prototype filter P(z) with small $\rho$ in (8) can be very expensive. In the following section, we will introduce the FRM technique to overcome this difficulty.

## 3. FRM TECHNIQUE

The basic structure of a filter synthesised using the FRM Technique was introduced in [16], which is shown in Fig. 2. From Fig. 2, we observe that the delay of the interpolated base linear phase filter F(z) in the upper branch is replaced by L delays. Then, it is cascaded with a masking filter $F_{Ma}(z)$. In the lower branch, the delay of the Complementary interpolated base filter $F_c(z)$ is replaced by L delays and then cascaded with a masking filter $F_{Mc}(z)$.

That is

$$P(z) = F(z^L)F_{Ma}(z) + F_c(z^L)F_{Mc}(z) \tag{14}$$

Where $|F(e^{jw})| + |F_c(e^{jw})| = 1 \tag{15}$

If F(z) is a linear symmetric FIR filter with odd order $L_F$, then we can choose $F_c(z)$ as

$$F_c(z) = z^{-(L_F-1)/2} - F(z)$$

There are two ways to choose the transition band of P(z) as illustrated in Fig. 3. From Fig. 3c and d, we see that the transition band of P(z) is provided either

by $F(z^L)$ or $F_c(z^L)$. The first case is referred to as Case A, whereas the second case as Case B. They are depicted in Fig. 3c and d, respectively. Note that the transition width of P(z) is narrower than that of F(z) by a factor of L. Clearly, the complexity of F(z) can be reduced by increasing the length of L. But this will increase the complexity of the two masking filters. If the transition width is not required to be too sharp when compared with the pass band, then it suffices to use the upper branch and discard the lower branch. This will significantly reduce the number of coefficients in the overall filter.
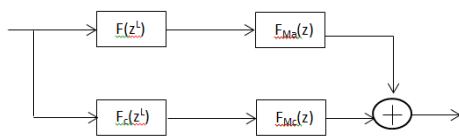


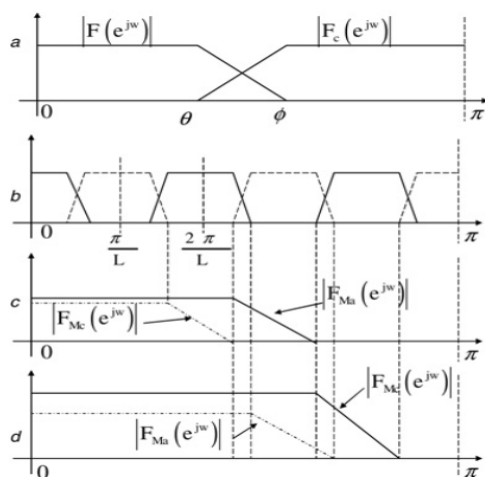Fig.2 Configuration of the design filter with sharp bands utilising FRM technique



Fig.3 Magnitude response functions in the FRM technique (a )Base filter (b)Interpolated base filter (c)Transition band of P(z) provided by $F_{Ma}(z)$ (d)transition band of P(z) provided by $F_{Mc}(z)$

Case 1 : L=kM (k ≥ 1 is an integer). From Fig. 3b, we see that $\pi/M = k\pi/L$ is the centre of the filter $F(e^{jw})$ or $F_c(e^{jw})$. However, 3 dB attenuation point of the prototype filter $P(e^{jw})$ is located at

$$w_{3dB} \quad \frac{\pi}{M}$$

and, one of the centre frequency of $F(e^{jw})$ or $F_c(e^{jw})$ transverses the frequency $w_{3dB}$ of the transition band of the desired prototype filter $P(e^{jw})$. Clearly, this case is unrealisable by the FRM technique.

Case 2: L = 2kM + k′ (k ≥ 1 is an integer and 1 ≤ k′< M). Since L = 2kM + k′ and 1 ≤ k′<, M, we have

$$\frac{2k\pi}{L} < \frac{\pi}{M} < \frac{(2k+1)\pi}{L}$$

In Fig. 3, we see that the transition band of $P(e^{jw})$ is determined by $F_c(e^{jw})$, that is, Fig. 3d is used. Let $w_p = (1-\rho)\pi/M$, $w_s = (1+\rho)\pi/M$, and let $\theta$ and $\phi$ be the pass band edge and the stop band edge of $F(e^{jw})$. Furthermore, let

$$m = \lceil w_s L/2\pi \rceil$$

where $\lceil w_s L/2\pi \rceil$ denotes the smallest integer larger than $w_s L/2\pi$. Then

$$\theta = 2m\pi - w_s L, \quad \phi = 2m\pi - w_p L$$

Case 3: L = (2k + 1)M + k′ (k ≥ 1 is an integer and 1 ≤ k′ < M). Since L = (2k + 1)M + k′, we have

$$\frac{(2k+1)\pi}{L} < \frac{\pi}{M} < \frac{(2K+2)\pi}{L}$$

Thus, the transition band of $P(e^{jw})$ is determined by $F(e^{jw})$. That is, Fig. 2c is applied. Let

$$m = \lfloor w_p L/(2\pi) \rfloor$$

where $\lfloor w_p L/(2\pi) \rfloor$ denotes the largest integer less than $w_p L/(2\pi)$. Then

$$\theta = w_p L - 2m\pi, \quad \phi = w_s L - 2m\pi$$

Case 4: M > L. Since M> L, $\pi/M < \pi/L$. In this case, only the upper branch of Fig. 2 is used. That is

$$P(z) = F(z^L)F_{Ma}(z) \quad (16)$$

In this case,

$$\theta = w_p L, \phi = w_s L$$

Suppose that the lengths of $F(z), F_{Ma}(z)$ and $F_{Mc}(z)$ are $L_F, L_{Ma}$ and $L_{Mc}$ and $L_{Ma} = L_{Ma}$, respectively. Let N be the total number of distinct coefficients of P(z). Then, $N = L_F + 2L_{Ma}$. Let

$$N_o = \frac{\Phi_1(w_p, w_s)}{2\rho/M}$$

where $\Phi_1(w_p, w_s)$ is given in Appendix of [16]. If $2\rho/M \leq 0.2$, then, $L_F \quad N_o/L$. From [16], the optimal L is given by

$$L_{opt} \quad \frac{1}{2}(2\rho/M)^{-1/2} \tag{17}$$

If $M < L_{opt}$, then either Case 2 or 3 can be applied.

If $M > L_{opt}$, Case 4 will be used. Otherwise, which

case is to be applied will be determined by the problem concerned.

If Case 2 or 3 is applied, then

$$N \quad 8N_o^{-1/2}\rho/M \tag{18}$$

If Case 4 is applied, then

$$N \quad \left(\frac{2}{L}-\frac{1}{M}\right)N_o \tag{19}$$

From (18) and (19), we see that if the transition width is sharp (i.e. $\rho$ is sufficiently small), the number of the distinct filter coefficients designed by the FRM technique is far smaller than those obtained by standard methods, such as the one reported in [16].

## 4. PROTOTYPE FILTER DESIGN

Suppose that $L_P - 1$ is a multiple of $M$. Then, the overall transfer function $T_O(e^{jw})$ of the filter bank can be simplified to

$$T_o(e^{jw}) = \frac{1}{D}e^{-jw(L_p-1)}\sum_{m=0}^{M-1}P(e^{jw}W_M^{-m})P(e^{-jw}W_M^m)$$

$$= \frac{1}{D}e^{-jw(L_p-1)}\sum_{m=0}^{M-1}|P(e^{j(w-(2m\pi/M))})|^2 \tag{20}$$

From (20), we know that there is no phase distortion in $T_O(e^{jw})$. According to (11), the overall aliasing transfer function $T_d(z)$ can be controlled by proper design of the stopband of $P(z)$.
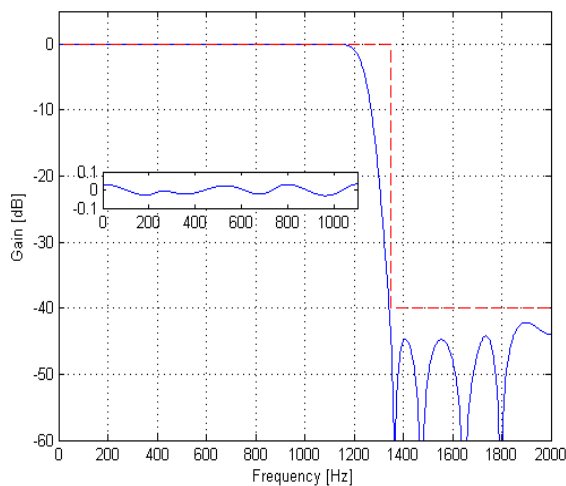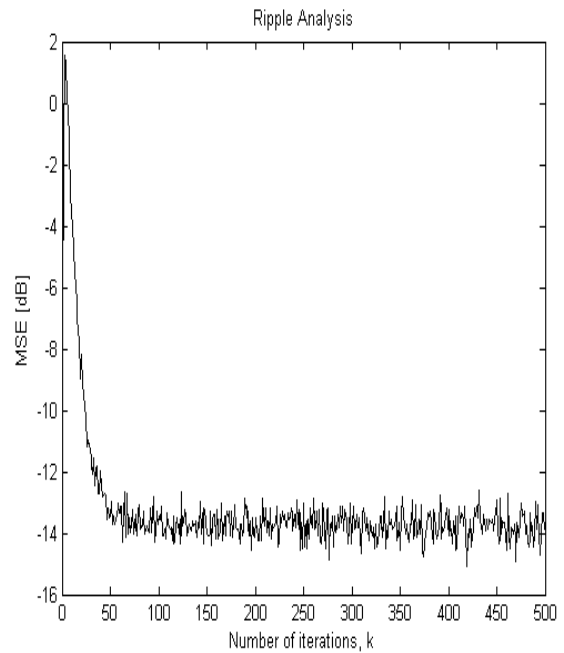
Fig. 4 The Magnitude response of the filter.

Fig.5 Ripple Analysis of the filter.

## 5.CONCLUSION

In this paper, we have developed a new method for the design of an FIR filter bank with a sharp transition band by utilising the FRM technique. First, the relationship between the interpolator in the FRM structure and the channel number of the filter bank was established. Then, the design of such a filter bank was formulated as a non-linear optimisation problem with continuous inequality constraints.

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